Moog Components Group manufactures a comprehensive line of brush-type and brushless motors, as well as brushless controllers. The purpose of this document is to provide a guide for the selection and application of DC motors and controllers. This document is not intended to be a design guide. The terms presented herein are simple and straightforward; the calculations are simplified but accurate. If more information is necessary, an applications engineer will be happy to answer any question or address any concern that may arise.

APPLICATIONS - WHAT TO CONSIDER

It is safe to say that no two applications are exactly the same. An apparatus that spins a pulley or fan is much different from one that precisely positions a workpiece or cutting tool. Regardless of how simple or complex the application, there are common requirements to consideration for the selection of the proper motor and / or controller. Some common considerations are: (presented here in the form of questions)

What output torque is required?
Motor torque is a combination of the internal torque losses $T_f$ (a function of motor design) and external torque load $T_L$. External torque load is a function of load inertia and load acceleration. This will be covered later.

What speed range is required?
How fast should the motor run when loaded and unloaded?

What space is available for the motor?
What length motor is required? What is the maximum motor diameter? Motor dimensions may be dictated by performance requirements.

What is the source of power for the motor?
Is the source AC or DC? What are the current limits of the source? What is the voltage range of the source?

Is there any special shaft and / or mounting requirements?
Does the shaft need a flat or a keyway? What length and diameter does the shaft need to be? Is a rear shaft extension required? (rear shaft extensions are necessary when encoders, brakes, etc. are added)

Are there any environmental considerations? Environmental considerations include:
• Temperature
• Humidity
• Shock and vibration

Is the motor “Heat Sunk”?
A motor can be heat sunked by mounting it on a mass of thermally conductive material. The material conducts heat away from the motor. Heat sinking has a dramatic effect on motor performance. Effective heat sinking increases the continuous output torque capability of the motor.

What are the expected shaft axial and radial loads? What is the expected velocity profile?
A velocity profile is a graph that shows how quickly the motor accelerates to rated speed, the time the motor runs at rated speed, and how quickly the motor decelerates to zero speed.

MOTOR TERMINOLOGY

Motor terminology can be confusing; especially all those darn subscripts ($K_T$, $K_p$, $T_f$, $R_p$, etc.) This section will define some basic motor terms as well as the more common subscripts. The units associated with the subscripts are also presented (both standard and metric).

1. **Terminal Voltage** – the voltage applied to the terminals of a motor.
2. **Peak Torque** – the maximum torque a motor can produce for short periods of time, before irreversible demagnetization of the motor’s magnets occurs. In smaller motors with higher resistance, the impedance of the motor often limits the peak torque.
3. **Rated (Continuous) Torque** – the maximum torque, at rated speed, the motor can produce on a continuous basis, without exceeding the thermal rating of the motor.
4. **Continuous Stall Torque** – the maximum torque, at zero speed, the motor can produce without exceeding its thermal rating.
5. **Rated Current** – the approximate amount of current the motor will draw at its rated torque point.
6. **Rated Speed** – the approximate motor speed at its rated torque point.
7. **Rated Power** – the maximum output power the motor can produce without exceeding its thermal rating. (output power is a function of speed and torque)
8. **Torque Sensitivity** – the relationship of output torque to the input current of the motor.
9. **Back EMF** – This is the ratio of generated output voltage to driven speed. (Also referred to as CEMF counter-electromotive force or generator voltage.)
10. **Terminal Resistance (Brushless DC Motors)** – static line to line resistance @ 25 deg. C.
11. **Terminal Resistance (Brush-Type DC Motors)** – dynamic resistance of the armature, brushes, and lead wires at a predetermined current @ 25 deg. C.
12. **Friction Torque** – the amount of torque required to overcome a motor’s static friction. Bearings, brushes, shaft seals, etc. all introduce friction into the motor.
13. **Thermal Resistance** – the ratio of a motor’s temperature rise to the motor’s power loss
14. **Motor Constant** – the ratio of a motor’s output torque to the motor’s input power. Motor constant is a figure of merit commonly used to compare motor capability.
15. **Speed / Torque Gradient** - the negative slope of the speed / torque line measured in rpm / oz-in.
COMMON SUBSCRIPTS

Conversions for some of the more commonly used motor parameters are:

- **KT** – Torque Constant
  \[ 1 \text{ Nm / amp} = 141.612 \text{ oz-in / amp} \]
  \[ 1 \text{ oz-in / amp} = 7.0615 \times 10^{-3} \text{ Nm / amp} \]

- **KE** – Back EMF Constant
  \[ 1 \text{ v / krpm} = 9.5493 \times 10^{-3} \text{ volt per rad / s} \]
  \[ 1 \text{ volt per rad / s} = 104.72 \text{ v / krpm} \]

- **KM** – Motor Constant
  \[ 1 \text{ oz-in / } \sqrt{\text{w}} = 7.0615 \times 10^{-3} \text{ Nm / } \sqrt{\text{w}} \]
  \[ 1 \text{ Nm / } \sqrt{\text{w}} = 141.612 \text{ oz-in / } \sqrt{\text{w}} \]

- **JR** – Rotor Inertia
  \[ 1 \text{ gm-cm}^2 = 1.14 \times 10^{-5} \text{ oz-in-s}^2 \]
  \[ 1 \text{ oz-in-s}^2 = 7.0615 \times 10^4 \text{ gm-cm}^2 \]

### BASIC MOTOR THEORY

Permanent magnet DC motors convert electrical energy into mechanical energy. This conversion takes place due to the interaction of the motor’s two magnet fields. One of these magnetic fields is created by a set of permanent magnets (on the brush-type motor, the stator usually contains the permanent magnets; the brushless motor’s magnets are a part of the rotor assembly). The other magnetic field is created by current flowing through the motor’s windings (the windings of a brush-type motor are contained in the armature (rotor), while the brushless windings are part of the stator assembly. In general, the stator is the stationary member of the motor, while the rotor is the rotating portion of the motor. The interaction of these two fields causes a resulting torque; the result of which is motor rotation. As the rotor turns, the current in the windings is commutated, resulting in a continuous torque output. (brush-type motors are mechanically commutated, while brushless motors are electronically commutated).

Three basic concepts must be understood when examining basic motor operation. These concepts are:

1. **Torque**
   Torque, also known as a moment of force, is a measure of the twisting effect that produces rotation about an axis. Simply stated mathematically, torque is the product of a force and the perpendicular distance from the pivot point to the force vector, or \( T = F \times D \). Typical units of torque are Nm, oz-in, ft-lbs, etc. The torque produced by a \( P_e = T \times S / 1352 \) (units: watts; oz-in, rpm) motor is the sum of internal torque losses (friction and windage – commonly labeled \( T_f \)) the external load torque (\( T_L \)). In a motor, the output torque is a function of the magnetic circuit, the number of magnet poles, and the number and configuration of the winding conductors. One of the two most important constants is that of torque sensitivity, or the torque constant (\( K_T \)). The output torque of a motor may be found by the following equation:
   \[ T = K_T \times I \] (units are: (SI) Nm; Nm / amp, amps (ENG.) oz-in; oz-in / amp, amps)
Example #1
Problem:
A C13-L19W10 has a torque constant of 3.42 oz-in / amp. The motor is drawing 1.5 amps. What is the output torque of the motor?
Solution:
The basic equation for motor output torque is \( T = K_t \times I \). \( T = 3.42 \text{ oz-in/amp} \times 1.5 \text{ amps} = 5.13 \text{ oz-in} \)

2. Speed
Motor speed is also a function of the magnetic circuit, the number of magnet poles, and the number and configuration of the winding conductors. The second important constant to be considered is the BACK EMF or voltage constant \( (K_e) \). This is an important constant, as it will determine the speed of a motor at a specified applied (terminal) voltage. The basic motor voltage equation is:
\( E_g = K_e \times n \) (units are: (SI) volts; volts / rad / sec, rad (ENG.) volts; volts / krpm, krpm

Speed is usually specified as either No-Load Speed or as Rated Speed. (see definitions is previous section). There is a useful, though greatly simplified equation that will allow you to get a rough estimate the no-load speed of a motor. This simplified equation is:
units are: (SI) rad / sec; volts, volts / rad / sec (ENG.) krpm; volts, volts / krpm

Example #2
Problem:
A BN34-25AF-01LH is to be operated at 24VDC. What will the approx. no-load speed be?
Solution:
The basic equation for a rough approximation of no-load speed is:

\[ N_{nl} = \frac{V_T}{K_E} = \frac{24 \text{ volts}}{3.10 \text{ volts/krpm}} = 7.742 \text{ krpm (7,742 rpm)} \]

Note that this simplified estimate does not include motor running losses.

3. Power
Power is defined as the rate of doing work. In dealing with motors, two units are typically used; watts and horsepower. When dealing with motor power, one must differentiate between input power and output power. Input power is the product of the voltage applied to the motor and the current drawn by the motor \( (P_i = EI) \). Output Power is a function of the motor’s speed and output torque. Output power may be calculated according to the following equations:
\( P_o = T \times \omega / 1352 \) (units: watts; Nm, rad / sec)

Efficiency is the ratio of output power to input power. Efficiency is calculated by:

Example #3
Problem:
A motor has a terminal voltage of 24 VDC. It draws 12.5 amps of current. The output torque is 120 oz-in at 2900 rpm. What is the input power? What is the output power? Determine the efficiency of the motor.
Solution:
\[ P_i = E I = 24 \text{ volts} \times 12.5 \text{ amps} = 300 \text{ watts} \]
\[ P_o = T \times \omega / 1352 = 120 \times 2900 / 1352 = 257.4 \text{ watts} \]
\[ \text{Eff.} = (\text{output power} / \text{input power}) \times 100 = 257.4 \text{ watts} / 300 \text{ watts} \times 100 = 85.8\% \]

SPEED-TORQUE CURVE
The relationship between speed and torque in brush-type and brushless motors is linear. A linear speed-torque curve is very desirable, especially in servo applications. A typical speed-torque curve is shown in Figure #1.
INCREMENTAL MOTION

A very common motor application is one in which the motor is accelerated from zero speed to operating speed in a certain time period, runs at speed for a period of time, and then decelerates to zero speed in yet another period of time. It is common to plot the speed and time values of a motor’s motion. This graphical representation is known as a velocity profile. A typical velocity profile is one in which the resulting motion “waveform” is trapezoidal.

Example #1

The above is a velocity profile for a brushless motor operating under closed-loop control. Metric units are used in working the example. We will solve for torque required for each time period, as well as RMS torque. The following motor and load parameters are:

Load Torque (T_L) – 1.0 Nm
Motor Friction (T_F) – 7.1 x 10^-2 Nm
Motor Inertia (J_M) – 1.7 x 10^-3 kg*m^2
Load Inertia (J_L) – 4.0 x 10^-4 kg*m^2

Step #1 – Find Acceleration and Deceleration (times t_1 and t_3)
In this example the motor accelerates from 0 rad / sec to 500 rad / sec in .250 sec. The motor decelerates from 500 rad / sec to 0 rad / sec in .250 sec. Therefore:
\[ a = \frac{Dw}{Dt} = \frac{500 \text{ rad / sec}}{.250 \text{ sec}} = 2000 \text{ rad/sec}^2 \]

Step #2 – Find Torque required for Acceleration (time t_1)
In general, Torque equals the product of inertia and acceleration, T = J·α. When the motor is accelerating, Torque = (inertia x acceleration) + friction

\[ T = (J_L + J_M) \cdot \alpha + (T_F + T_L) \]
\[ T = (4.0 \times 10^{-4} \text{ kg.m}^2 + 1.7 \times 10^{-3} \text{ kg.m}^2)(2000 \text{ rad/sec}^2) + (1.0 \text{ N.m} + 0.071 \text{ N.m}) \]
\[ T = 4.200 \text{ N.m} + 1.071 \text{ N.m} \]
\[ T = 5.271 \text{ N.m} \]

Step #3 – Find Torque required for Constant Velocity (time t_2)
When the motor is not accelerating or decelerating, the required torque is equal to the sum of the motor’s friction torque and the load torque. The equation is:

\[ T = (T_F + T_L) \]
\[ T = (0.071 \text{ N.m} + 1.0 \text{ N.m}) \]
\[ T = 1.071 \text{ N.m} \]

Step #4 – Find Torque required for Deceleration (time t_3)
When the motor is decelerating, the friction introduced by the motor and load are subtractive; or simply stated, Torque = (inertia x acceleration) - friction

\[ T = (J_L + J_M) \cdot \alpha - (T_F + T_L) \]
\[ T = (4.0 \times 10^{-4} \text{ kg.m}^2 + 1.7 \times 10^{-3} \text{ kg.m}^2)(2000 \text{ rad/sec}^2) - (1.0 \text{ N.m} + .071 \text{ N.m}) \]
\[ T = 4.200 \text{ N.m} - 1.071 \text{ N.m} \]
\[ T = 3.129 \text{ N.m} \]
Step #6– Find RMS Torque

RMS torque is calculated using the following equation and data:

\[
T_{\text{rms}} = \sqrt{(T_1)^2 t_1 + (T_2)^2 t_2 + (T_3)^2 t_3} / (t_1 + t_2 + t_3)
\]

\[
T_{\text{rms}} = \sqrt{(5.271)^2(0.250) + (1.071)^2(2.1) + (3.129)^2(0.250)} / (0.250 + 2.1 + 0.250)
\]

\[
T_{\text{rms}} = 2.131 \text{ N.m}
\]